Control

13/10/2015 الثلاثاء د. محمد عرفة محاضرة [2]

Root Locus :-

الحل الهندسي للجذور

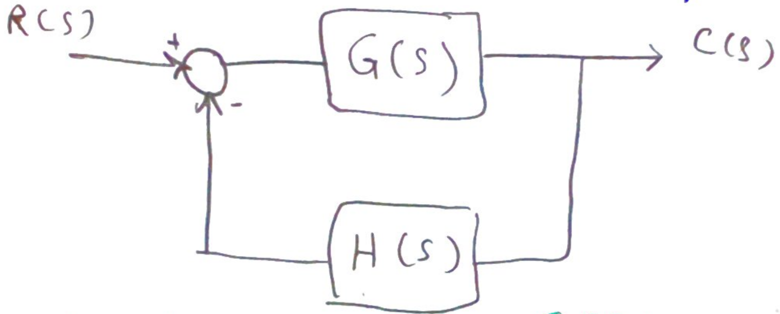
Stability

① Algebric methods
- Routh method

Graphical methodes
- Root Locus
- Bode Diagram
- polar plot
- Nyquist

we use it for Designin in control system

Root Locus: The Locus of the roots of ch. eqn (poles)



C.L.T.F = closed loop T.F. $= \dfrac{G(S)}{1+GH(S)}$

O.L. T.F = open loop T.F. $= GH(S)$

ch. eqn $\Rightarrow 1 + GH(S) = 0 \Rightarrow$

مسميات لجذور المعادلة

① the roots of ch. equation
② " system poles
③ " " eigen values

Ex: ch. eqn: $1 + GH(S) = 0$

$$S^3 + 3S^2 + 4S + k = 0$$

find the range of k that make the system Stable

$\Longrightarrow$ Turn over

تعريف آخر

Root Locus:-

The locus of the roots of ch. eqn (Poles) that depend on variable parameter (K) that takes positive values $(0 \longrightarrow \infty)$

Ex: given $\text{O.L.T.F.} = \dfrac{K}{S(S+1)}$

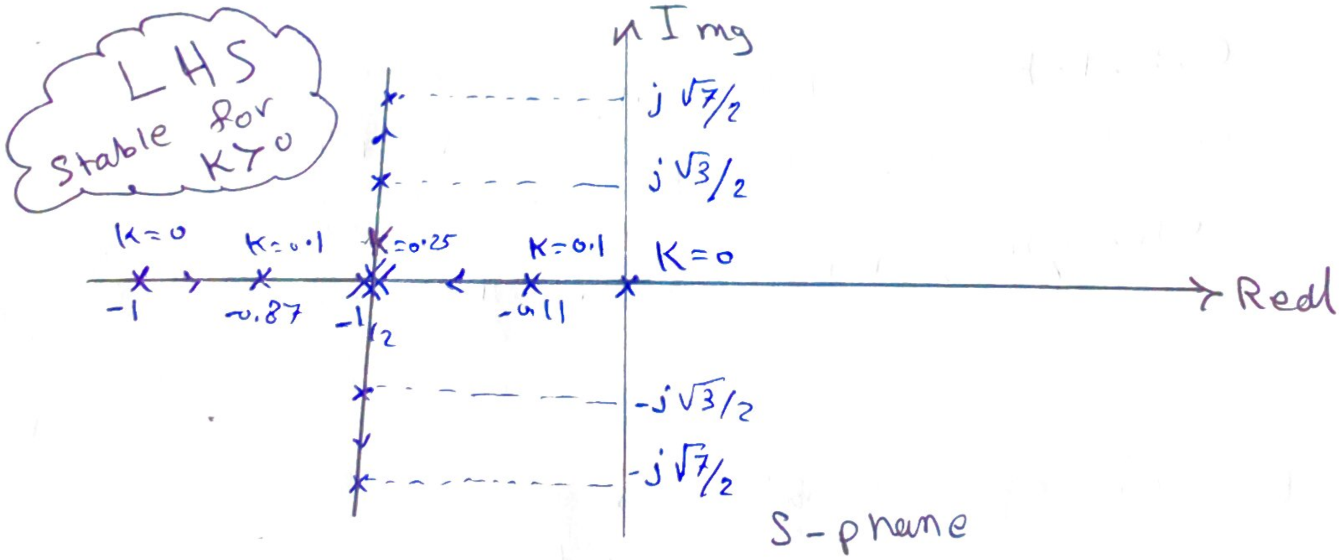
Solution: ch. eqn $\Rightarrow 1 + GH(S) = 0$

$$1 + \frac{K}{S(S+1)} = 0 \Rightarrow S(S+1) + K = 0$$

$$\boxed{S^2 + S + K} \text{ ch. eqn}$$

$$S_{1,2} = \frac{-1 \pm \sqrt{1-4K}}{2}$$

K	0	0.1	0.25	1	2
$S_{1,2}$	0, -1	-0.11, -0.88	$-\frac{1}{2}, -\frac{1}{2}$	$-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$	$\frac{-1}{2} \pm j\frac{\sqrt{7}}{2}$



الطريقة التالية تقريبية للابتعاد
عن كثرة الحسابات

$\Longrightarrow$ Turn over

Ex: $GH(S) = \dfrac{K(S+1)}{S(S+3)(S+4)}$

Draw the root Locus and find the range of K that make the system stable

Solution:

لازم ال O.L.T.F. يكون معطى

Step ① O.L. poles ⇒ 3 poles ⇒ 0, −3, −4

O.L. Zeros ⇒ 1 zero ⇒ −1

step ② S-plane:

Poles ⟶ x

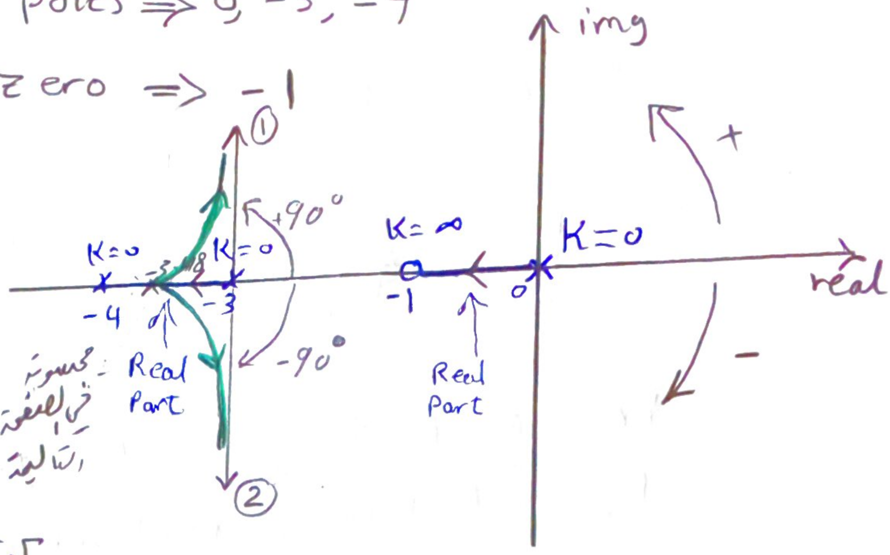
Zero ⟶ o

Poles ⇒ $x^{K=0}$

end at zero ⇒ $o^{K=\infty}$

at K=0 ⇒

⇒ O.L.T.F. = C.L.T.F



step ③ Real Part:

① 0 ⟶ −1 , ② −3 ⟶ −4

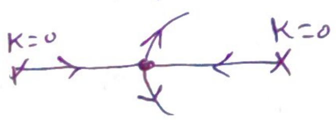
step ④ Breaking Point:

هي نقطة تقع على المحور الحقيقي تكون فيها ال K أكبر أو أصغر ما يمكن

① Break away Point

نقطة هروب

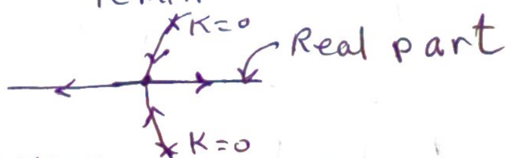
K max



② Breakin Point

نقطة تلاقي

K min



Ch. equation = $1 + GH(S) = 0$ = root locus eqn

$$GH(S) = -1$$

$$\frac{K(S+1)}{S(S+3)(S+4)} = -1 \Rightarrow K = -\left[\frac{S(S+3)(S+4)}{(S+1)}\right]$$

$$\frac{dK}{ds} = 0$$

$$-\left[\frac{(S+1)(3S^2+14S+12)-(S^3+7S^2+12S)(1)}{(S+1)^2}\right] = 0$$

$$(S+1)(3S^2+14S+12)-(S^3+7S^2+12S) = 0$$

$$S^3+5S^2+7S+6=0 \Rightarrow S_{1,2,3} = -3.48,\ -0.75 \pm j1.0\ldots$$

$\therefore$ Breaking Point at $S_b = -3.48$

$$K_b \Big|_{S_b=-3.48} = -\left[\frac{s(S+3)(S+4)}{(S+1)}\right] = 0.35$$

breaking point at -3.48, $K_b = 0.35$

$K_b \rightarrow$ at breaking point, repeting poles

* two poles at the same point

Step ⑤ Asymptotes:

① no. of asympt $= \underset{\text{no. of poles}}{n} - \underset{\text{no. of zeros}}{m}$

$= 3 - 1 = 2$

② $\sigma_c = \dfrac{\sum \text{Poles} - \sum \text{Zeros}}{n-m}$

center of asymptotes

$$\sigma_c = \frac{(0-3-4)-(-1)}{2} = \frac{-7+1}{2} = \frac{-6}{2} = -3$$

③ $\theta = \dfrac{(2L+1)180}{n-m}$, $L = 0, 1, 2, 3, 4 \Leftarrow$ كرر بعدد الخطوط

$L=0 \Rightarrow \theta_1 = +90°$

$L=1 \Rightarrow \theta_2 = 270° = -90°$

the system is stable for all $K > 0$

- you can check by routh array.

Ch. eq$\underline{n}$: $1 + GH(s) = 0$

$$\Rightarrow 1 + \frac{K(s+1)}{s(s+3)(s+4)} = 0$$

$$s(s+3)(s+4) + K(s+1) = 0$$

$$s^3 + 7s^2 + 12s + Ks + K = 0$$

$$s^3 + 7s^2 + (12+K)s + K = 0 \Leftarrow \text{Ch. eq}\underline{n}$$

Routh array:

s^3	1	$12+K$
s^2	7	K
s^1	$\frac{7(12+K)-K}{7} > 0$ ①	
s^0	$K > 0$ ②	

$\Rightarrow 7(12+K) - K > 0$

$84 + 7K - K > 0$

$6K > -84$

$K > -\frac{84}{6} \Rightarrow K > -14$

from ① and ②

$\Rightarrow$ The range for stability $K > 0$

Ex 2: $GH(s) = \dfrac{K}{s(s+2)(s+5)}$

① O.L poles $\Rightarrow 0, -2, -5$

O.L. Zeros $\Rightarrow$ X

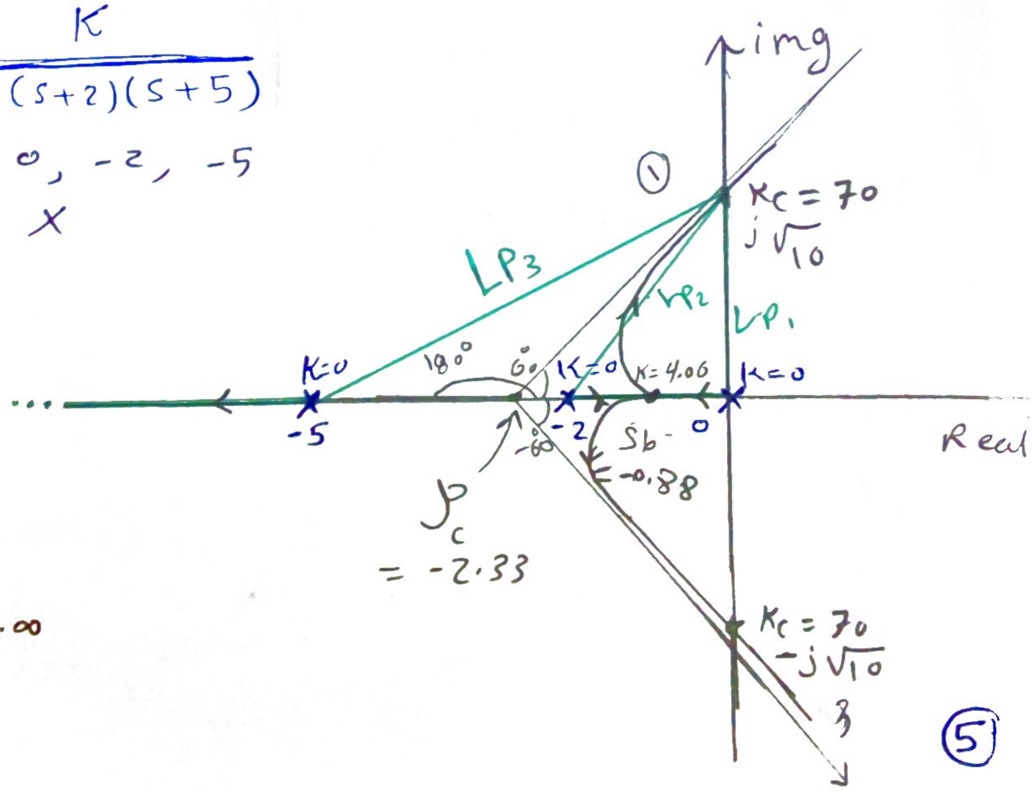
② s-plane

Pole $\rightarrow$ x ($K=0$)

Zero $\rightarrow$ o ($K=\infty$)

③ Real Part $\rightarrow 0 \rightarrow -2$

$\rightarrow -5 \rightarrow -\infty$



④ Breaking Point

ch. eq $\Rightarrow 1 + GH(s) = 0$

$GH(s) = -1$

$$\frac{K}{s(s+2)(s+5)} = -1 \Longrightarrow \boxed{K = -[\overbrace{s(s+2)(s+5)}^{s^3+7s^2+10s}]}$$

$$\frac{dK}{ds} = -[3s^2 + 14s + 10] = 0$$

$$3s^2 + 14s + 10 = 0$$

$$\Rightarrow s_{1,2} = -0.88, -3.78$$

Accepted, neglected

$s_b = -0.88$

$$K_b\Big|_{s_b=-0.88} = -[s(s+2)(s+5)]$$

$$\Rightarrow K_b = 4.06$$

⑤ Asymptotes:

① no. of asymptots $= n - m = 3 - 0 = 3$

② $$\sigma_c = \frac{\Sigma \text{poles} - \Sigma \text{zeros}}{n-m} = \frac{(0-2-5)-(0)}{3}$$

$$\sigma_c = \frac{-7}{3} = -2.33$$

③ $$\theta = \frac{(2L+1)\cancel{180}\ 60}{\cancel{n-m}\ \cancel{3}\ 1}$$

$L = 0 \Rightarrow \theta_1 = 60$

$L = 1 \Rightarrow \theta_2 = 180$

$L = 2 \Rightarrow \theta_3 = 300 = -60$

⑥ at img axi's — نقطة إضافية بسبب تقاطع ال asymptotes مع المحور الرأسي (احسب قيمة و عند أي K يحدث التقاطع)

Ch. eqn $\Rightarrow 1+GH(s)=0$

$$s(s+2)(s+5)+K=0$$

$$s^3+7s^2+10s+K=0$$

using routh array

$$\begin{array}{c|cc} s^3 & 1 & 10 \\ s^2 & 7 & K \\ s^1 & \frac{70-K}{7} & \\ s^0 & K & \end{array}$$

$A(s) \Rightarrow$ (s^2 row)

صف فردي $\rightarrow$ (s^1 row)

$\frac{70-K}{7} > 0$ ① $\Rightarrow K < 70$

$K > 0$ ② $\Rightarrow K > 0$

the range for stability

$$0 < K < 70$$

at $K = 70$

the row $s^1 \Rightarrow$ has zero values

$$\underset{\text{critical}}{K_c} = 70$$

$$A(s) = 7s^2 + K_c = 0$$

$$7s^2 + 70 = 0 \Rightarrow s^2 = -10 \Rightarrow s = \pm j\sqrt{10}$$

K at point S_o located at the root locus:

$$K\Big|_{S_o} = \frac{\prod \text{Poles}}{\prod \text{zeros}}$$

$$K\Big|_{s_0} = \frac{L_{p_1} \cdot L_{p_2} \cdot L_{p_3}}{1}$$

في حالة كانت على مقياس

L_{p_i} توصيل كل نقطة بكل الـ poles وحسب أطوال الخطوط

L_{z_i} 〃 〃 〃 〃 الـ zeros 〃 〃 〃

لو لم يوجد خطوط نضع $\Pi\, zeros = 1$

Report: أثبت القانون السابق

مساعدة: ابدأ بال Ch. eqn

Ex3:- $GH(s) = \dfrac{K(s+4)}{s(s+2)}$

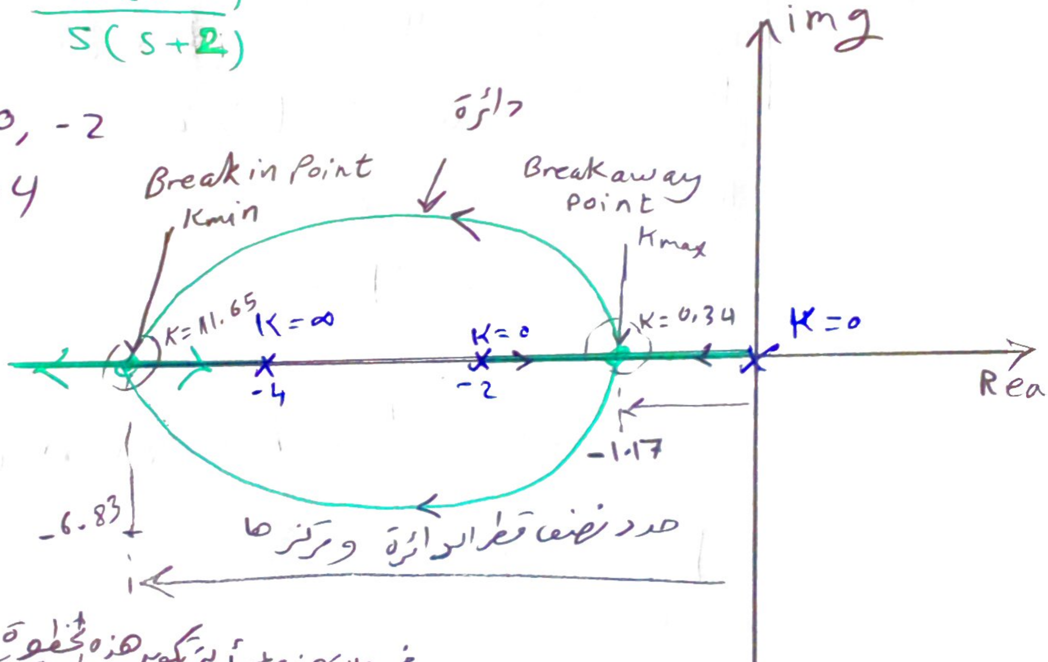
① O.L. poles ⇒ 0, −2

O.L. zeros ⇒ −4

② s-plane

Pole $\longrightarrow \overset{K=0}{\times}$

zero $\longrightarrow \overset{K=\infty}{o}$



③ Real Part

① $0 \longrightarrow -2$

$-4 \longrightarrow -\infty$

④ Asymptotes في حالة هذه مثلاً لا تكون هذه الخطوة ضرورية

① No. of asy. $= n - m = 2 - 1 = 1$

② $\sigma_c = \dfrac{(0-2)-(-4)}{1} = 2$

③ $\theta = \dfrac{(2L+1)\,180}{n \neq m}$

$L = 0 \Rightarrow \theta = 180°$

④ Breaking Points

Ch. eqn: $1 + GH(s) = 0 \Rightarrow GH(s) = -1$

$$\frac{K(s+4)}{s(s+2)} = -1$$

$$K = -\left[\frac{s(s+2)}{(s+4)}\right]$$

$$\frac{dK}{ds} = -\left[\frac{(s+4)(2s+2) - (s^2+2s)(1)}{(s+4)}\right] = 0$$

$\therefore (S+4)(2S+2) - (S^2+2S) = 0$

$S^2 + 8S + 8 = 0$

$S_{1,2} = -1.17 \ \& \ -6.83$

Break away   Breakin

Breaking points

① $S_{b_1} = -1.17 \Rightarrow K_{S_{b_1}} = 0.34$

② $S_{b_2} = -6.83 \Rightarrow K_{S_{b_2}} = 11.65$

for the circle

$r = \dfrac{(6.83 - 1.17)}{2} = 2.83$ نصف قطر الدائرة

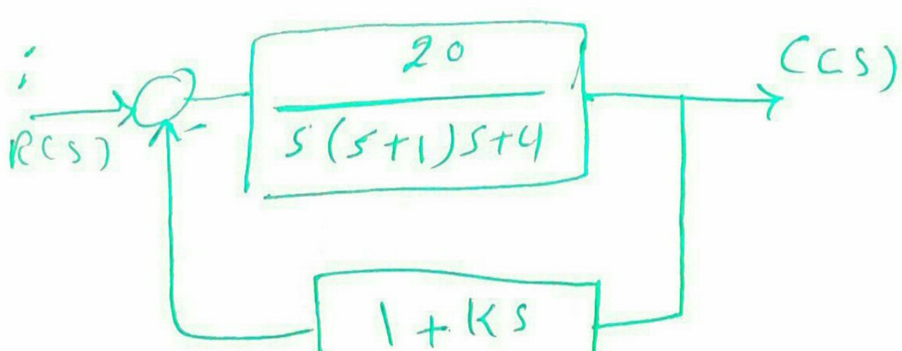
$\underset{\text{center}}{C} = -2.83 - 1.17 = -4$

هذا النوع الوحيد من الـ كل المطلوب بالرسم

بمقياس (إلا إذا يكون هناك متطلبات أخرى)

system is stable for all $K > 0$

Report;



Sketch the root Locus and find the range of K for stability

ملاحظة: ال K لازم تكون مش مجموعة مع
رقم أو مضروبة في رقم

$$GH(S) = \frac{(K+3)\,S}{(S+3)(S+5)}$$

$\Leftarrow$ ارجع للصورة $1 + GH(S)$

وحاول تفصل ال K في طرف لوحدها

$$GH(S) = \frac{20K\,(S+4)}{(S+3)(S+5)}$$

بدل ال 20K بمتغير آخر مثل (K') وحل المسألة

ويبقى الكلام على K' أي 20K